ABSTRACT

Real-time sea state measurements could be used to greatly enhance the safety and efficiency of offshore operations. In this paper, a variational data assimilation scheme is developed to improve ocean wave field estimates from marine navigation radars. The assimilation scheme minimizes a cost function which is defined as the difference between radar observations and predictions from a nonlinear wave evolution model over a time interval which is referred to as assimilation interval. An adjoint method is used to calculate the gradient of the cost function with respect to control variables, and the descent direction for minimization is obtained from tangent linear model. The proposed assimilation scheme is validated using synthetic and field data.

KEY WORDS: Variational assimilation; adjoint method; offshore; minimization; marine radars; wave model.

INTRODUCTION

Many of offshore operations can be carried out only if the wave heights remain at a reasonably low amplitude for the duration of the operation and the period of the swell is sufficiently different from the natural periods of oscillation of the vessels involved. During such operations, decisions need to be made shortly before or during the task about suspension of work based on expected risk of damage or loss of life. These decisions must be based upon accurate short and medium-term wave forecasts. Recent advances in remote sensing techniques allow for measurement of the ocean wave fields using marine radars in near-real time. Assimilation of real-time observations into a short-term wave forecasting model can greatly enhance the safety and efficiency of offshore operations. Given an estimate of the present state of the sea surface (initial condition), wave forecast models simulate the short-term evolution of the sea surface subject to the imposed boundary conditions. The quality of the forecast is strongly dependent on the accuracy of both initial/boundary conditions. Given the imperfections in numerical models as well as inherent noise in radar observations and the complexity of the nonlinear radar imaging mechanisms, data assimilation is considered as an extremely important tool to improve estimates of the sea state.

In the past two decades, an increasing interest has been shown in the application of conventional marine navigation radars for imaging the sea surface. Oudshoorn (1960), Ijima et al. (1964), Wright (1964), Willis and Beaumont (1971), Evmenov et al. (1973), and Mattie and Haris (1979) were among the first to report the use of such radars for imaging ocean surface waves. The fundamental interaction between radar and sea surface is assumed to be Bragg scattering. The longer surface gravity waves become visible by four different mechanisms: hydrodynamic modulation of the short waves by the long waves, shadowing effect, tilting modulation, wind drift, and strain effect of long waves’ orbital velocity.

Young and Rosenthal (1985) developed a technique for a full time series of radar images, each taken at a successive revolution of the radar antenna, and using these series of images, found a three dimensional energy density spectrum through a three dimensional Fourier transform in wave number-frequency space. Ziemer and Rosenthal (1987) proposed a modulation transfer function from the radar wave spectrum to surface wave spectrum in Fourier space, based on tilting, shadowing and hydrodynamic effects, and found out that nonlinear influence in transfer function is mainly due to shadowing. Nieto Borge (1999) used X-Band marine radars to find the characteristics of the sea state such as directional spectrum and significant wave height.

Real-time assimilation of radar measurements needs a fast and efficient numerical wave model that can still capture basic nonlinear interactions. Pseudo-spectral models are increasingly used to model surface waves and can meet both these criteria. Pseudo-spectral methods for free surface problems were initially inspected using the time-dependent Fourier-series method of Fenton and Rienecker (1982), where instead of using a boundary integral formulation, the solution of the three dimensional Laplace equation was written in terms of Fourier series. Series truncation was the only approximation in this method. It has been shown that free surface boundary conditions can be reduced to a pair of evolution equations for the two free surface variables (Zakharov, 1968). This closed set of equations is written in two horizontal coordinates and therefore the dimension of the problem is reduced by one. Efforts to use this approach have been made by West et al. (1987), Matsumo (1992), Craig and Sulem (1993), and Choi (1995) among others.

Although a number of physics-based modulation transfer functions (MTF) have been proposed to relate the radar backscatter intensity to the sea surface elevation, it is still difficult to obtain reliable estimates of the wave field due to the amount of noise present in the radar data and an inadequate understanding of all radar imaging mechanisms. On the other side, wave models are always far from perfect, so data
assimilation could potentially improve the estimates of the wave field from marine radars, by combining radar measurements and model predictions. A variational assimilation scheme will be developed in this paper in which all observations over the assimilation interval will be used. A cost function is defined as the difference between radar measurements and model predictions with appropriate weights over the assimilation interval. We look for an optimal initial/boundary conditions that minimize the cost function through an iterative procedure, each step of which requires the explicit knowledge of the gradient of the cost function with respect to initial/boundary conditions. The gradients are calculated using adjoint technique (Le Dimet and Talagrand, 1986), while the Newton descent direction for the minimization stage is obtained from tangent linear model. The iteration repeats until the magnitude of the gradient is decreased to a pre-defined value. The assimilation scheme will be validated using synthetic data as well as field data collected off a ship in Alaska in April, 2006.

WAVE MODEL FORMULATION

The problem of surface water waves was formulated by Zakharov (1968) as a Hamiltonian system that conserves mass, momentum, and energy, with Hamiltonian function defined as:

\[ E = \frac{1}{2} \int \left[ \nabla \phi \cdot \nabla \phi + \phi^2 \right] \, dz \, dx + \frac{1}{2} g \eta^2 \, dx \]

(1)

Where \( \phi(x, z, t) \) and \( \eta(x, t) \) are velocity potential and surface elevation respectively, \( x = (x, y) \), and \( \nabla \) stands for gradient in horizontal plane. Surface waves can be written in the form of Hamilton’s canonical equations for surface elevation \( \eta(x, t) \) and velocity potential at the free surface, \( \Phi(x, t) = \phi(x, \eta, t) : \)

\[ \eta_t = \frac{\partial E}{\partial \Phi} \quad \text{and} \quad \phi_t = -\frac{\partial E}{\partial \eta} \]

(2)

For an ideal fluid, neglecting the effect of surface tension and surface pressure, free surface boundary conditions can be written in terms of surface elevation and free surface velocity potential as:

\[ \eta_t + \nabla \Phi \cdot \nabla \eta = \left( \nabla \eta \right) W \]

(3)

\[ \Phi_t + \frac{1}{2} \left[ \nabla \Phi \cdot \nabla \eta \right]^2 + g \eta = \left( \nabla \eta \right) W^2 \]

(4)

Where \( W \) is the vertical velocity at the free surface, \( \Phi \) and \( W \) can be expanded in Taylor series about \( z = 0 \):

\[ \Phi(x, t) = \phi_0 + \eta \phi_0 - \frac{\eta^2}{2} \nabla^2 \phi_0 + \frac{\eta^3}{6} \nabla^3 \phi_0 + \frac{\eta^4}{24} \nabla^4 \phi_0 + O(\varepsilon^5) \]

(5)

\[ W(x, t) = w_0 - \eta \nabla^2 \phi_0 - \frac{\eta^2}{2} \nabla^2 \phi_0 + \frac{\eta^3}{6} \nabla^3 \phi_0 + \frac{\eta^4}{24} \nabla^4 \phi_0 + O(\varepsilon^5) \]

(6)

Where, \( \varepsilon = k \eta \) is the wave steepness parameter, and the Laplace equation has been used to substitute \( -\nabla^2 \phi_0 \) for \( \frac{\partial^2 \phi_0}{\partial z^2} \). To derive the evolution equations for tangential velocity at the free surface and surface elevation, we first take the gradient of Eqs. 4 and 5 and substitute \( \phi_0 \) for \( \nabla \phi \), and \( u \) for \( \nabla \Phi \):

\[ u_s(x, t) = u_s + \nabla (\eta \phi_0) - \left( \frac{\eta^2}{2} \nabla \eta \right) u_s - \frac{\eta^3}{6} \nabla^2 \phi_0 + \left( \frac{\eta^4}{24} \nabla^4 \phi_0 \right) + O(\varepsilon^5) \]

(7)

\[ W_s(x, t) = w_0 - \eta \nabla^2 \phi_0 - \frac{\eta^2}{2} \nabla^2 \phi_0 + \frac{\eta^3}{6} \nabla^3 \phi_0 + \frac{\eta^4}{24} \nabla^4 \phi_0 + O(\varepsilon^5) \]

(8)

Where, \( u_s \) is the tangential velocity at the mean free surface and \( \eta \) and \( \phi_0 \) are free surface tangential velocities in the \( x \) and \( y \) directions respectively. The relationship between two physical variables at the mean free surface, \( u_s \) and \( \phi_0 \), can be found by solving the linear Dirichlet boundary value problem in the lower half plane using the Fourier transform method which yields:

\[ w_s(x, t) = L[u_s] \]

(9)

Where the integral operator \( L \) for one dimensional case and finite depth \( h_0 \) can be written as:

\[ L[u_s] = \frac{1}{2h_0} \mathcal{P} \int_{-\infty}^{\infty} \frac{u_s(x, t)}{\sinh[(\pi / 2h_0)(x - x')] \sinh[(\pi / 2h_0)(x' - x)]} \, dx' \]

(10)

Which is the Hilbert transform for deep water. Operator \( L \) acting on a Fourier component is given by:

\[ L[e^{ikx}] = -i \tanh(kh_0) e^{ikx} \]

(11)

Where, \( k = (k_x, k_y) \), and \( k = \sqrt{k_x^2 + k_y^2} \). Using the relationship between \( W_0 \) and \( u \), the series (7) can be inverted to any order of wave steepness. Up to fourth order we have:

\[ u_s(x, t) = u_s - \nabla (\eta L[u_s] - \eta L[\nabla (\eta L[u_s])] - \nabla (\eta \nabla (\eta L[u_s]))]) \]

(12)

By substituting Eq. 12 into Eq. 8, and substituting for \( W \) in Eqs. 3 and 4, one can derive a system of equations for the evolution of surface elevation and tangential velocity at the free surface, though for numerical implementation, a finite number of terms in the series will be kept depending on the desired accuracy. Up to third order approximation can be written as:

\[ \eta = L[u_s] - \nabla \cdot (\eta u_s) - L[\nabla (\eta L[u_s])] + \nabla (\frac{1}{2} \eta^2 L[u_s]) \]

(13a)
\[ u_w = -g\nabla \eta - \frac{1}{2} \nabla (u \cdot u) + \frac{1}{2} \nabla (L[u], L[u]) - \nabla (\eta L[u], u) \quad (13b) \]
\[ -\nabla (L[u], [\nabla (\eta L[u])]) + O(\varepsilon^4) \]

To solve the system (13) numerically, a pseudo-spectral method using a Fourier basis in space (Fornberg and Watham, 1978) is used in which free surface variables, \( \eta \) and \( u_s \), are expanded in Fourier series as:

\[ \eta(x, t) = \sum_{n=-N/2}^{N/2-1} \sum_{m=-M/2}^{M/2-1} \hat{\eta}_{nm}(t) e^{iK_n x + iK_m y} \]
\[ u_s(x, t) = \sum_{n=-N/2}^{N/2-1} \sum_{m=-M/2}^{M/2-1} \hat{u}_{nm}(t) e^{iK_n x + iK_m y} \]  

(14)

Where \( N, M \) are the number of Fourier modes in \( x \) and \( y \) directions, respectively, and \( K_n = 2\pi / L_x \) and \( K_m = 2\pi / L_y \), with \( L_x \) and \( L_y \) being the computational domain lengths in the \( x \) and \( y \) directions respectively. Fourier coefficients \( \hat{\eta}_{nm}(t) \) and \( \hat{u}_{nm}(t) \) are computed by two dimensional Fast Fourier Transform for given \( \eta \) and \( u_s \). The differentials and linear transforms are evaluated in Fourier space as:

\[ \nabla(\hat{\eta}_{nm}(t)e^{Kx}) = iK\hat{\eta}_{nm}(t)e^{Kx} \]
\[ L[\hat{\eta}_{nm}(t)e^{Kx}] = -i \tanh(K[h_0])\hat{\eta}_{nm}(t)e^{Kx} \]

(15)

Where \( K = (nK_x, mK_y) \) and \( |K| = \sqrt{(nK_x)^2 + (mK_y)^2} \). Nonlinear terms are evaluated in physical space, and the evolution Eqs. 13 are integrated in time using a fourth order Runge-Kutta scheme. To prevent aliasing error from happening in nonlinear model, a simple low-pass filter is used that sets the amplitudes of the spectrum to zero for wave numbers larger than a cut-off wave number. The total energy is monitored carefully to minimize the energy loss from filtering process.

ASSIMILATION SCHEME

A variational assimilation scheme is used to minimize the difference between model predictions and marine radar observations over an interval. The assimilation is performed in physical space, and the initial sea surface elevation is chosen as control variable. Cost function is defined as:

\[ J(\eta_0) = \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{M} \left[ \int_{x_{k-1}}^{x_k} \int_{y_{j-1}}^{y_j} \left( \eta(x, y, t_k) - \eta_{obs}(x, y, t_k) \right)^2 dx dy \right] \]

(16)

Where \( N \) and \( M \) are the numbers of modes in \( x \) and \( y \) directions respectively, and \( N_{obs} \) is the number of radar observations in the assimilation interval. The goal is to find the optimal initial condition of the sea surface, \( \eta_0 \) that minimizes the cost function over the interval \([t_0, t_1]\), though the cost function is not directly related to the initial condition, \( \eta_0 \). Following Le Dimet and Talagrand (1986), gradient of the cost function with respect to initial condition can be calculated using adjoint technique in which starting from the initial condition \( \eta_0 \) and \( u_{s,0} \), the forward Eqs. 13 are first integrated from \( t_0 \) to \( t_1 \). Then, starting from the difference between model-predicted value and observations at time \( t_i \), the adjoint equations are integrated backward in time from \( t_i \) to \( t_0 \) being forced, at observation times, with the difference between predicted values and measured values. The value of adjoint variable at time \( t_0 \) will give us the desired gradient. This gradient is then used in a minimization algorithm specialized for large-scale problems. In the following section we will derive the adjoint system of equations for Eqs. 13.

Adjoint Model

Gradient of the cost function (16) with respect to initial condition can be calculated by one forward integration of the Eqs. 13, and one backward integration of the adjoint system. To derive the adjoint model, Eqs. 13 are first linearized with respect current state of the system. The linearized system is referred to as “Tangent Linear Model (TLM)” which models the evolution of initial perturbations in time. TLM equations corresponding to Eqs. 13 up to third order can be written as:

\[ \eta' = L[u'] - \nabla \cdot (\eta' u' + \eta u') - L[\nabla (\eta L[u']) + \eta L[u'] + \nabla (\frac{1}{2} \eta' L[u'] + \eta L[u']) + O(\varepsilon^5) \]
\[ + L[\nabla (\eta L[u'] + \eta L[u'] + \eta L[u'] + \eta' L[u'] + \eta' L[u']) + \nabla (\frac{1}{2} \eta' L[u'] + \eta' L[u']) + O(\varepsilon^4) \]

(17a)

\[ u'_s = -g\nabla \cdot (u' \cdot u') + L[u'] + \nabla (\eta L[u'] + \eta L[u'] + \eta L[u'] + \eta' L[u'] + \eta' L[u']) \]
\[ - \nabla (\eta' L[u'] + \nabla \cdot u' + \eta L[u'] + \eta L[u'] + \nabla \cdot u') - \nabla (\eta L[u'] + \eta L[u'] + \eta L[u']) + \nabla (\frac{1}{2} \eta' L[u'] + \eta' L[u']) + O(\varepsilon^4) \]

(17b)

To derive the adjoint equations, we first find the inner product of the assumed adjoint variables \( \tilde{\eta} \) and \( \tilde{u}_s \) with TLM Eqs. 17. The inner product in Euclidian space is defined as:

\[ \langle \tilde{\eta}, u_s \rangle = \int_t \int_x \tilde{\eta} \cdot u_s \, dx \, dt \]

(18)

After integrating by parts to transfer derivatives from \( \eta' \) and \( u'_s \) to \( \tilde{\eta} \) and \( \tilde{u}_s \), and assuming periodic boundary conditions for forward and adjoint equations, all boundary terms will vanish, and we will be left with two sets of equations for the evolution of adjoint variables \( \tilde{\eta} \) and \( \tilde{u}_s \), which should be integrated backward in time in order to be stable. For simplicity, we write the first order adjoint equations here:

\[ \tilde{\eta}_t = -g \nabla \cdot \tilde{u}_s \]

(19a)

\[ \tilde{u}_{s,t} = L[\tilde{\eta}] \]

(19b)

Adjoint Eqs. 19 are also solved using pseudo-spectral method. For simplicity we use the first order adjoint equations (19) for the assimilation purpose. The adjoint equations are solved backward in time, starting from the error at the terminal time and being forced with the errors at observation times. The value of the adjoint variable \( \tilde{\eta} \) at \( t = 0 \) will give us the gradient of the cost function with respect to \( \eta_0 \).

Since it is assumed that the surface elevation is the only observed
variable, the value of tangential velocity at the free surface at the beginning of integration is found using linear approximation and the same is done for the adjoint variable $\bar{u}$. The calculated gradient will then be used in an iterative minimization method which will be introduced in the following section. An overview of the proposed adjoint assimilation scheme is given in Fig. 1.

Fig. 1. Overview of Assimilation Scheme.

**Minimization Method**

The feasible methods for such large-scale unconstrained minimization problem are (a) the limited memory conjugate gradient method (Shanno and Phua, 1980); (b) quasi-Newton type algorithms (Gill and Murray, 1972); (c) limited memory quasi-Newton methods such as LBFGS algorithm (Liu and Nocedal, 1989); (d) truncated Newton algorithms (Wang, Navon, Zou, and LeDimet, 1995), and (e) adjoint Newton algorithm (Wang, Droegemeier, and White, 1998). Methods (a) – (d) find the descent direction using the gradient of the cost function, while the adjoint Newton algorithm provides a new approach to find the Newton descent direction by integrating the tangent linear model backward in time. This algorithm has a quadratic convergence rate. Adjoint Newton method will be implemented in this paper and its performance will be discussed in the results section. A schematic presentation of adjoint and tangent linear approaches is given in Fig. 2.

Fig. 2. Schematic presentation of adjoint and tangent linear approaches in finding descent directions.

**Adjoint Newton Method.** Adjoint Newton method or tangent linear approach was proven (Wang et al., 1998) to be a robust method for large-scale assimilation problems. In this method, starting with an initial guess $\eta_0$, gradient of the cost function with respect to initial condition is calculated using adjoint technique by integrating the adjoint system (19) backward in time as illustrated in Fig. 1.

$$g_\epsilon = g(\eta_\epsilon) = DJ(\eta_\epsilon)$$

(20)

Subscript $k$ represents the iteration number. A Newton line search direction is obtained by integrating the tangent linear model (17) backward in time, starting from the difference between predicted and measured values at the final time and being forced with errors at observation times.

$$d_\epsilon = -\eta'(0)$$

(21)

The value of the control variable is then updated:

$$\eta_{k+1} = \eta_k + \alpha_k d_k$$

(22)

Where $\alpha_k$ is the step size obtained by doing a line search:

$$J(\eta_k + \alpha_k d_k) = \min J(\eta_k + \alpha d_k)$$

(23)

Line search is conducted using Davidon’s cubic interpolation, which satisfies the following Wolfe conditions:

$$J(\eta_k + \alpha_k d_k) \leq J(\eta_k) + \beta \alpha_k \nabla \eta_k^T d_k$$

(24a)

$$g^T_k (\eta_k + \alpha_k d_k) d_k \leq \beta$$

(24b)

Where $\beta = 0.0001, \beta = 0.9$, and $T$ is the transpose sign. Given a tolerance criterion, $\epsilon$, iteration is repeated until convergence is achieved:

$$\|g_{k+1}\| \leq \epsilon \|g_k\|$$

(25)

Where $g_0$ is the value of the gradient at the beginning of minimization.

The numerical cost for integrating the tangent linear model is similar to that required for integrating the adjoint model. This algorithm does not have storage limitation problem since it does not require the calculation of either Hessian or its inverse.

**NUMERICAL EXPERIMENTS**

To assess the feasibility and efficiency of the proposed assimilation scheme, two experiments are carried out as follows:

1) Simulated observations using JONSWAP 2D power spectrum with random errors.

2) Field measurements from X-Band marine radar, conducted off a ship in Alaska in April, 2006.

**2D Synthetic Data**

Two dimensional wave field was generated by a JONSWAP power spectrum. In order to prepare the initial wave field it is necessary to consider a directional frequency spectrum $S(f, \theta) = P(f)g(\theta)$, where $P(f)$ is the original JONSWAP spectrum and then to transform it into
the associated wave number spectrum, \( S(k_x, k_y) \). The directional spreading function \( G(\theta) \) used here, is defined as:

\[
G(\theta) = \begin{cases} 
\frac{1}{\varphi} \cos^2 \left( \frac{\pi}{2\varphi} \theta \right) & \text{if } |\theta| \leq \varphi \\
0 & \text{otherwise}
\end{cases}
\]  

(26)

\( \varphi \) is a parameter that provides a measure of the directional spreading. Using the linear dispersion relation \( \omega = \sqrt{\kappa^2 \tanh(\kappa h)} \), wave number spectrum can be written as:

\[
S(k_x, k_y) = \frac{\alpha_{\omega_{\gamma}}}{2|\kappa|} e^{-\frac{1}{2}|\kappa|} \times \exp \left[ \frac{-(\kappa^2 - \kappa_0^2)^2}{2\sigma_{\omega_{\gamma}}^2} \right] \times G(\theta)
\]  

(27)

Where \( \theta = \arctan \left( \frac{k_y}{k_x} \right) \). Water depth was assumed to be 100m.

Two dimensional surface elevation is calculated in the following way:

\[
\eta(x, y) = \sum_{i=1}^{N} \sum_{j=1}^{M} C_{ij} \cos(k_x x + k_y y + \phi_i)
\]  

(28)

Where the \( \phi_i \)'s are uniformly distributed random phases on the interval \([0, 2\pi]\), and \( C_{ij} = \sqrt{\Delta S(k_x, k_y)} \) \( k_0 \), where \( k_1, k_2 \) were defined before. The numerical simulation was performed in a field of length \( 1530m \times 1530m \) with 256 Fourier modes in each direction, and a dominant wave number \( k_0 = 0.0443m^{-1} \), corresponding to a characteristic wave length of \( \approx 142m \), and the peak period of \( T_p \approx 9.53s \). The adopted time step was 0.25s. Generated clean initial condition is plotted in Fig. 3. After generating the initial condition for \( \varphi = 3.3^\circ, \alpha \gamma = 3^\circ, \alpha = 0.0663 \) \( (H_s = 3.25m) \), \( \sigma_{\omega_{\gamma}} = 0.07 \) for \( |k| \leq k_0 \), and \( \sigma_{\omega_{\gamma}} = 0.09 \) for \( |k| > k_0 \), it was evolved for 100s and 8 frames with 2.5 second intervals were picked out. Pseudo-observations then were generated by adding 40% uncorrelated Gaussian noise to the chosen frames. Noisy initial condition is plotted in Fig. 4. The synthetic sea surface is rougher in this case as expected.

![Fig. 3. Generated initial condition using JONSWAP 2D power spectrum.](image)

![Fig. 4. Noisy initial condition generated by adding 40% Gaussian noise to clean model-generated sea surface.](image)

The Newton descent direction calculated from tangent linear approach and the gradient calculated from adjoint technique were compared with finite difference approximations. In finite difference method, each component of the control variable (initial condition) is perturbed by a small value \( \delta \), and the gradient is approximated as follows:

\[
D_J(\eta_0) = \frac{J(\eta_0 + \delta) - J(\eta_0)}{\delta}
\]  

(29)

The calculated gradient from this method is sensitive to the value of \( \delta \), and can be a very poor estimate when \( \delta \) is not chosen properly. The numerical cost of this approach is also high, because for a problem with \( (N \times M) \) variables, the forward model should be integrated \( (N \times M) \) times while in adjoint (tangent linear) method, the same gradient (or the descent direction) can be obtained by two integrations of the forward and adjoint (tangent linear) model respectively. Three methods mentioned above, are compared in Figs. 5~8. The value of \( \delta \) was \( 10^{-10} \) in this simulation. In Figs. 5~6, the gradient calculated from adjoint technique is compared with finite difference approach, as well as tangent linear descent direction, along two perpendicular lines.
Contour lines of the gradients from adjoint method has been compared with finite difference approach in Fig. 7, while the descent direction obtained from tangent linear method has been compared with finite difference in Fig. 8. The shown 16 by 16 box was extracted from the lower left corner of the computational domain.

After verifying the gradient and descent direction numerically, assimilation experiment was conducted with the cost function being minimized using the adjoint Newton method described among the lines 20–25. The first observation frame was used as initial guess. The descent direction at each iteration was normalized with its initial value to improve the minimization performance. Value of the step length $\alpha$ at the beginning of each iteration was 10 and this value was reduced to satisfy the Eq. 24. The method converged after 11 iterations through which the cost function reduced to about half of its initial value, as shown in Fig. 9. To ensure that the converged solution is a global minimum, the assimilation was repeated with several randomly perturbed initial guesses, and the difference in the converged solution was not significant.

Reconstructed initial condition is shown in Fig. 10.

To show the ability of assimilation to remove the noise, the converged initial condition was evolved and first 50 seconds of surface elevation time histories were compared with clean solution as well as noisy solution in Figs. 11–12. It should be noted that assimilation interval in the present experiment was 17.5 s. It is observed that the sea state estimate is significantly improved using the proposed data assimilation technique.
Comparing with Dispersion Filter. Performance of the proposed data assimilation scheme is compared with a linear dispersion filter acting on the noisy synthetic data. We first find the three dimensional Fourier transform of the noisy record (the same 8 frames which were used for the assimilation experiment) in wave number-frequency space, and then multiply it by a Gaussian filter function defined as:

$$F(i, j, l) = \exp \left[ -0.25 \left( \frac{\omega_i - \omega_k}{f_\omega} \right)^2 \right]$$

(30)

Where $\omega_i$ is the angular frequency obtained from linear dispersion relation $\omega_i = \sqrt{k_x + \tan h(k_y, k_z)}$, $\omega_k = \text{ld} \omega$ , and $d\omega = 2\pi/T$ ($T$ is the assimilation interval). $f_\omega$ is the filter band width, which is taken to be the maximum frequency (i.e. $f_\omega = \frac{N_{\omega} \omega}{2} d\omega$). After transforming back to physical space, we compared the reconstructed initial condition from the filter with the one obtained from data assimilation, which is shown in Fig. 13 along a line in y-direction. The time histories of the surface elevation are also compared in Fig. 14. It is observed that even in the present linear case, filtering is not able to recover the clean solution, and data assimilation still works better.

Field Data

Field experiments were conducted to evaluate the ability of both Doppler (coherent) and non-coherent X and S-Band radars to measure ocean wave fields and provide data for validation of the data assimilation and wave forecast models. The experiments were carried out at a site located approximately 70 miles southeast of Kodiak, Alaska, aboard a ship in April, 2006. Triaxys buoy as well as GPS-based directional wave buoys were used to provide ground-truth data on the directional wave field around the ship. During this experiment, radar backscatter data from the ship’s X-Band navigational (non-coherent, HH polarized) radars were digitally recorded, an example of which is given in Fig. 15.
The small square and triangle in the picture show the location of the wave buoys. Wave field retrievals were performed initially using a “tilt modulation” approach similar to that of Dankert and Rosenthal (2004), as well as simulated data sets, in order to improve the match between radar and buoy retrieved wave fields. Preliminary results of comparison between radar and buoy retrieved wave field (spectrum and directionality) is given in Fig. 16. Investigation on different retrieval methods and ability of data assimilation methods to improve the radar estimates for this experiment is still the subject of current research.

![Fig. 16. Preliminary comparison between radar and buoy retrieved wave fields.](image)

CONCLUSIONS

An efficient variational assimilation scheme has been developed that is able to improve the sea state estimates from marine navigation radars in near-real time, through combining a limited number of radar images with a fast and effective numerical wave forecast model. Initial condition was taken as control variable in the numerical experiments. The gradient of the cost function with respect to initial sea state was calculated using adjoint method, while Newton descent direction for minimization was obtained using tangent linear approach with similar numerical cost as forward and adjoint integrations.

Unconstrained minimization plays a major role in the total numerical cost of the assimilation process. Gradients and descent directions in the proposed method are obtained with just two additional integrations of adjoint and tangent linear models respectively, instead of \( M \times N \) times in direct finite difference method.

Data assimilation technique was able to significantly remove the noise in 2D synthetic data, and performed better than linear dispersion filter. In linear case, both numerical model and dispersion filter solve the same system of equations, however the model’s time step is much finer that the time interval between successive observation frames. Therefore, even for the linear model, dispersion filter does not work as efficient as the assimilation does. It is believed that higher order models will mandate the use of data assimilation.

Preliminary results of comparing radar and buoy retrieved wave profiles have been presented, and it is believed that the proposed data assimilation method with appropriate boundary conditions will be able to improve the match between the radar and buoy retrievals. This is the subject of the authors’ ongoing research.

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